LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2015**

MT 2503 - ANALY. GEOM. OF 3D, FOURIER SERIES & NUM. THEORY

Date : 04/09/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

PART – A

ANSWER ALL QUESTIONS (10 x 2 = 20)

1. Find the distance of the origin from the plane $6x - 3y + 2z - 14 = 0$.
2. Find the angle between the planes: $2x + 4y - 6z = 11$ and $3x + 6y + 5z + 4 = 0$.
3. Find the equation of the sphere which has its centre at the point $(6,-1,2)$ and touches the plane $2x - y + 2z - 2 = 0$.
4. Find the equation of the sphere whose centre is at (2,3,0) and which passes through (1,0,2).
5. Define even and odd functions.
6. Find the number and sum of all the divisors of 360.
7. Find the number of integers less than n and prime to it when n=729 and 720.
8. State Fermat's theorem.
9. Prove that if a, b, c are positive and not equal, then (a+b+c) (bc+ca+ab)>9abc.
10. Show that $n^n > 1.3.5\ldots (2n-1)$.

PART – B

ANSWER ANY FIVE QUESTIONS (5 x 8 = 40)

11. a) Find the equation of the plane through the points (2,5,-3), (-2,-3,5) and (5,3,-3).
    b) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1,\ 2x + 3y + 4z - 7 = 0$ and perpendicular to the plane $x - 5y + 3z = 5$. (4+ 4)

12. Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0,\quad x + 2y + 3z = 8$ and touch the plane $4x + 3y = 25$.

13. Find the condition that the line $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$, where $l^2 + m^2 + n^2 = 1$ should touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

14. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series with period 2π, to be valid in the interval 0 to 2π.
    Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

15. If $f(x) = \begin{cases} -x \text{ in } -\pi < x < 0 \\ x \text{ in } 0 \le x < \pi \end{cases}$, expand $f(x)$ as Fourier series in the interval $-\pi$ *to* π. Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$

16. a) Find the highest power of 3 dividing 1000!
    b) Show that $x^5 - x$ is divisible by 30. (4 + 4)

17. Show that $\left(x^m + y^m\right)^n < \left(x^n + y^n\right)^m,\quad if\ m > n.$

18. Show that if $s = a_1 + a_2 + ... + a_n$, $\frac{s}{s-a_1} + \frac{s}{s-a_2} + ... + \frac{s}{s-a_n} > \frac{n^2}{n-1}$

unless $a_1 = a_2 = ... = a_n$.

PART – C

ANSWER ANY TWO QUESTIONS (2 x 20 = 40)

19. a) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}; \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

b) A plane passes through a fixed point (a,b,c) and cuts the axes in A, B, C. Show that the locus of the

centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2.$ (10+10)

20. a) A function f(x) is defined within the range $(0, 2\pi)$ by the relations $f(x) = \begin{cases} x & in\ (0,\pi) \\ 2\pi - x\ in\ (\pi, 2\pi) \end{cases}$.

Express f(x) as a Fourier series in the range $(0, 2\pi)$.

b) If $f(x) = \begin{cases} x & when\ 0 < x < \frac{\pi}{2} \\ \pi - x\ when\ x > \frac{\pi}{2} \end{cases}$,

expand f(x) as a sine series in the interval $(0, \pi)$. (10+ 10)

21. a) Show that $13^{2n+1}+9^{2n+1}$ is divisible by 22.
b) Show that (18)!+1 is divisible by 437. (8+12)

22. a) Prove that $8xyz < (y+z)(z+x)(x+y) < \frac{8}{3}(x^3 + y^3 + z^3)$.

b) State and prove Weirstrass inequalities. (10 + 10)

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